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Toward the energy and the system size dependence of elliptic flow: working on flow fluctuations

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Abstract.   In this talk I concentrate on two topics closely related to the understanding of the energy and the system size dependence of the elliptic flow: determination of the elliptic flow (v_2) "free" from the effects of non-flow and flow fluctuations, and the role of fluctuations in the initial eccentricity of the overlap region. I introduce a new approach for the analysis of the distribution in flow vector, dP/dq_n, namely, I propose to use the Bessel Transform of this ditribution. I show that the Bessel Transform method is similar to the Lee-Yang Zeroes method, and is very transparent in its meaning and applications.

1. Introduction

Anisotropic flow, and, in particular, elliptic flow, plays a very important role in our understanding of heavy ion collisions at high energies. Since the discovery of the in-plane elliptic flow at AGS [1], and later at CERN SPS [2], the attention to this phenomena has been continuously increasing. The first RHIC measurements [3] by the STAR Collaboration made it one of the most important observable at RHIC [4]. The large elliptic flow, along with its mass dependence at low transverse momenta [5, 6] are often used as a strong argument for early thermalization of the system; the observed [7] constituent quark scaling [4, 8] of elliptic flow strongly suggests that the system spends significant time in the deconfined state.

In order to understand deeper the physics of the elliptic flow and the processes governing the evolution of the system, one has to measure and understand elliptic flow dependence on the system size and collision energy. Having in mind also the dependence on the centrality of the collision makes the problem indeed 'multi-dimensional'. In [9] the authors suggested that the elliptic flow follows a simple scaling in the initial system eccentricity and the particle density in the transverse plane, $v_2/\varepsilon \propto 1/S \, dN_{ch}/dy$ (where ε is the initial system eccentricity and S is the area of the overlap region of two nuclei). If true, all the data taken at different

energies, colliding different nuclei, and at different centralities should collapse on the same universal curve. Indeed the available data are consistent with such a scaling [4, 10]. Unfortunately, the systematic uncertainties in the results are too large to conclude on how well the scaling holds. The main difficulties are the evaluation/elimination of the so-called non-flow correlations (azimuthal correlations not related to the reaction plane orientation), and effects of flow fluctuations, along with uncertainties in the calculation of the initial system eccentricity needed for such a plot.

The non-flow contribution is usually suppressed when one uses multi-particle correlations to measure anisotropic flow [11, 12]. Unfortunately, the measurements of flow via correlations of different number of particles are also different in their sensitivity to the event-by-event flow fluctuations[12,13]. Thus it become difficult to disentangle different contributions (non-flow and flow fluctuations) from measured azimuthal correlations. It was concluded in [14] that the Lee-Yang Zeroes method is the best for suppression of non-flow correlations. Initially I thought that this method could be less sensitive to the flow fluctuations. Below I show that this hope was over-optimistic, and the results of the Lee-Yang Zeroes method are affected by flow fluctuations in a similar way as four-particle cumulant results.

As it was already mentioned, the flow measurements via correlations of different number of particles, such as two- and four-particle correlation results, $v_2\{2\}$ and $v_2\{4\}$, are sensitive to flow fluctuations in different ways. The contribution of fluctuations should be taken out before one tries to explore the scaling $v_2/\varepsilon \propto 1/S\, dN_{ch}/dy$. Here we discuss a different and more convenient approach, namely to rescale the flow results with an appropriate value of eccentricity that already include the effect of fluctuations. Such type of eccentricities are studied and presented in the second part of this talk.

2. Fourier and Bessel Transforms of dP/dq_x and dP/dq^2

Consider the distribution in x component of n-th harmonic flow vector

$$Q_x = \sum_{i=1}^{M} \cos(n\phi_i) \equiv q_x \sqrt{M}, \qquad (1)$$

where M is the multiplicity of particles used to define the flow vector. Let us denote the distribution in Q_x in the case of no collective flow ($v_n = 0$) to be $f_{\mathrm{nf}}(x = Q_x)$. In the case of large multiplicities M, this distribution can be approximated by Gaussian, but we will not need an explicit form here. In the case of non-zero flow the distribution can be written as a superposition of "no-flow" distributions, f_{nf}, "shifted" in the direction of flow by appropriate amount (for details, see [16]; the necessary condition is $\sqrt{M} \gg 1$):

$$f(Q_x) \equiv \frac{dP}{dQ_x} = \int \frac{d\Psi}{2\pi} f_{\mathrm{nf}}(Q_x - vM\cos(n\Psi)). \qquad (2)$$

Now let us consider the Fourier transform of this distribution:

$$
\begin{aligned}
\tilde{f}(k) &= \langle e^{ikQ_x} \rangle = \int dQ_x e^{ikQ_x} \int \frac{d\Psi}{2\pi} f_{\rm nf}(Q_x - vM\cos(n\Psi)) \\
&= \int \frac{d\Psi}{2\pi} \int dQ_x e^{ikQ_x} f_{\rm nf}(Q_x - vM\cos(n\Psi)) \\
&= \int \frac{d\Psi}{2\pi} e^{ikvM\cos(n\Psi)} \int dt e^{ikt} f_{\rm nf}(t) = J_0(kvM)\tilde{f}_{\rm nf}(k)
\end{aligned}
\tag{3}
$$

Remarkably, the flow contribution is completely factorized out. As one expects the function $\tilde{f}_{\rm nf}(k)$ to be close to Gaussian (at large M it must approach Gaussian according to the Central Limit Theorem), the zeros of the Fourier transform will be determined by the zeroes of the Bessel function $J_0(kvM)$, thus providing a method of extraction of flow from the distribution in Q_x. For example, if k_1 is the first zero of the Fourier transform,

$$
v = j_{01}/k_1 M
\tag{4}
$$

where $j_{01} \approx 2.045$ is the first zero of J_0 Bessel function. The above result is the same as one would obtain applying the Lee-Yang Zeroes method (using "sum" generating function); in fact the relation to the Fourier transform was already pointed out in the original paper [14].

Even more interesting result one obtains considering two-dimensional distribution, $d^2P/dQ_x dQ_y$. In this case

$$
\begin{aligned}
\tilde{f}(k) &= \int dQ_x e^{ik_x Q_x} dQ_y e^{ik_y Q_y} \frac{d^2P}{dQ_x dQ_y} \\
&= \int dQ\, J_0(kQ) \frac{dP}{dQ} \ \sim\ J_0(kvM),
\end{aligned}
\tag{5}
$$

which, as shown above, is reduced to the Bessel transform of the distribution in the magnitude of the flow vector. Note that in this approach (valid in the limit of $\sqrt{M} \gg 1$) the flow contribution is completely decoupled from all other correlations. It happens due to the collective nature of flow. Note also that in the same limit one expects the distribution in flow vector to be Gaussian due to the Central Limit Theorem, thus explaining why the fitting of the distribution to the form derived in [16] (such fits have been used in [1, 12]) is also non-sensitive to non-flow correlations. Thus in this limit all three methods, the Bessel Transform, Lee-Yang Zeroes, and fitting q-distribution become very similar, if not equivalent.

Fig. 1 (left panel) shows the fit to the q-distribution of the simulated data (four million events with 400 particles in each event; 50% events have flow $v = 0.04$ and 50% with $v = 0.06$). Right panel shows the Bessel transform of the same distribution. One finds that the results obtained with two methods are indeed very close to each other. For a discussion of a small deviation from 0.05, expected for this case, see below.

Though our goal here is to calculate the integrated flow, it is straight-forward to generalize the method for the calculation of differential flow as well. For that
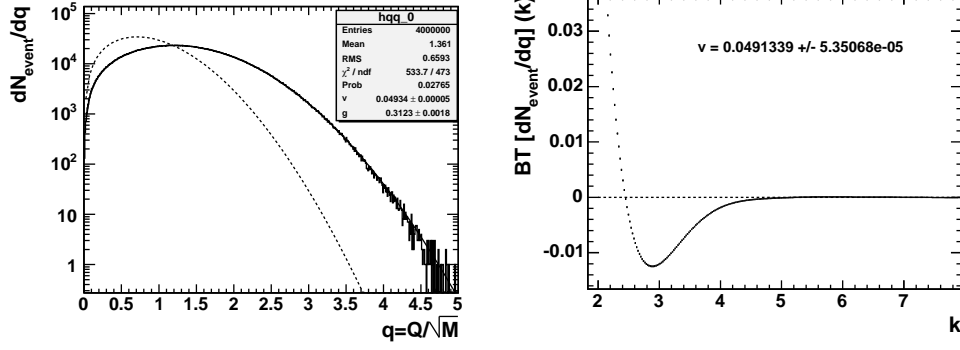
Fig. 1. (left panel) q-distribution for simulated events shown together with fit function. Dashed line shows the expected distribution in case of $v = 0$. (right panel) Bessel transform of the distribution on the left. (See the last row in Table 1 for the flow values obtained in this case).

one considers

$$\langle\cos(\phi_i)\, e^{ikQ_x}\rangle = \int \frac{d\Psi}{2\pi}d\phi_i(1 + 2v\cos(\phi_i - \Psi))\cos(\phi)e^{ikQ_x}f(Q_x)$$

$$= \int \frac{d\Psi}{2\pi}v\cos(\Psi)e^{ik\hat{v}M\cos(\Psi)}\tilde{f}_{\rm nf}(k) \;=\; vJ_1(k\hat{v}M)\tilde{f}_{\rm nf}(k), \tag{6}$$

where we have introduced notation $\hat{v}$ for the average flow in the region of the flow vector definition. From the above equation one can obtain the flow v by different methods. The one suggested in [14] is based on the relation

$$\langle Q_x e^{ikQ_x}\rangle = \hat{v}MJ_1(k\hat{v}M)\tilde{f}_{\rm nf}(k). \tag{7}$$

Combining Eqns. 6 and 7 one finds:

$$v = \hat{v}M\frac{\langle\cos(\phi_i)e^{ikQ_x}\rangle}{\langle Q_x e^{ikQ_x}\rangle}. \tag{8}$$

Alternative way would be to use the expression 3 directly. Then

$$v = \frac{\langle\cos(\phi_i)e^{ikQ_x}\rangle}{\langle e^{ikQ_x}\rangle}\frac{J_0(k\hat{v}M)}{J_1(k\hat{v}M)} \tag{9}$$

More detailed discussion of the differential flow measurements using Bessel Transform is deferred for later publication.

The main advantage of using the Bessel Transform (as well as Lee-Yang Zeroes method, or fitting the Q-distribution, as all three methods are very close in their assumptions and limitations) is that it is much less sensitive to the non-flow contribution compared to the standard method [15]. The sensitivity of these methods

to flow fluctuations is not that clear from the above equations. As it follows from the simulation results presented in Table 1, the actual sensitivity to fluctuations is very similar to that of four-particle cumulant approach.

Table 1 present the results obtained by different methods on simulated data without flow fluctuations, 100% events with $v = 0.04$ or $v = 0.06$, and 50/50 mixture of the above. Last three rows present results from simulations including large non-flow correlations (simulated by pairs of particles with erectly the same azimuth). The following observations follow: $v\{2\}$ results, as expected are sensitive to both non-flow effects and flow fluctuations (the presented results coincide within statistical errors with analytical expectations). $v_2\{4\}$, $v_2\{BT\}$, and $v_2\{q\text{-dist}\}$ are not affected by non-flow. For the case of 50/50 mixture of events with different flow values, $v_2\{4\}$ yields value below 0.05 due to flow fluctuations. What was somewhat unexpected is that the sensitivity (bias) of $v_2\{BT\}$, and $v_2\{q\text{-dist}\}$ to flow fluctuations is very similar to that of $v_2\{4\}$. More detailed examination of the Eq. 3 and expansion of the Bessel function around j_{01} up to the second order confirms this observation made on simulated data. This is also illustrated in Fig. 2 (read caption).

3. Eccentricity and eccentricity fluctuations

If the initial conditions are totally isotropic – no anisotropic flow is possible. So one concludes that at small initial anisotropies the final anisotropic flow must be *linearly* proportional to the initial spatial anisotropy of the system. Although it is not clear what one should use (what is most relevant) for the higher harmonic flow, in case of the elliptic flow it is relatively straight-forward: one should come up with some measure that differentiate the system geometry along the impact parameter, in-plane, and, correspondingly, out-of-plane, such as $(R_y - R_x)/(R_y + R_x)$ or $(R_y^2 - R_x^2)/(R_y^2 + R_x^2)$. where radii R_x and R_y are some characteristics of the system in- and out-of- the reaction plane directions. Note, that different definitions

Table 1. Flow values (in %) obtained by different methods.

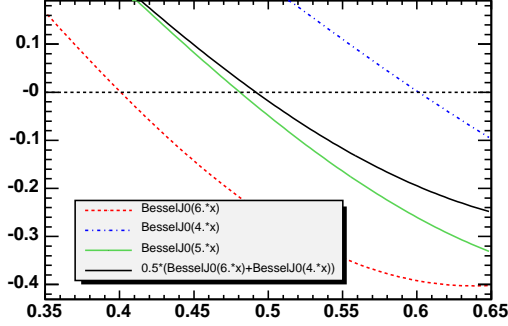|	$v_2\{2\}$	$v_2\{4\}$	$v_2\{BT\}$	$v_2\{q\text{-dist}\}$
without non-flow				
100% $v = 6.0$	6.005±0.003	6.006±0.006	6.018±0.004	5.999±0.002
100% $v = 4.0$	3.995±0.004	3.977±0.002	4.013±0.008	3.997±0.003
50/50 $v = 4$ & $v = 6$	5.100±0.004	4.895±0.007	4.908±0.004	4.920±0.004
including non-flow				
100% v=6.0	6.495±0.003	6.005±0.007	6.028±0.004	6.033±0.004
100% v=4.0	4.703±0.004	4.013±0.002	4.011±0.002	4.038±0.009
50/50 v=4 & v=6	5.670±0.005	4.905±0.008	4.913±0.005	4.933±0.005

Fig. 2. The most left and most right curves shows the expectations for Bessel Transform of $q-$distribution for 4% and 6% flow. The solid black line shows the average of these two. The green line shows the expectations for 5% flow. The difference in zeroes of black and green lines indicates the effect of flow fluctuations.

of the eccentricity give different values, but this is not a problem if one uses the *same* definition for all systems/centralities/experiments used in a comparison, and as long as the eccentricity itself is small. Recall that the goal of this exercise is to exploit the fact that at small eccentricities the final flow should be proportional to eccentricity. (Note that such proportionality would not at all be trivial if eccentricity is large). Having said all that, the most often used definition of eccentricity is

$$\varepsilon \equiv \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}, \tag{10}$$

where the x direction is taken along the direction of the impact parameter, and the average, with some weight, is taken over all points of the system. For the weight one can use the participating nucleon or produced particle densities, entropy or energy densities, density of binary collisions, etc.. A priory it is not known what weight would be the best to use, but again, note that at small eccentricities it should not matter provided the *same* weight is used for all the data.

In the standard *optical Glauber* definition of the eccentricity the weight function is assumed to be the same for all events with the same impact parameter; any event-by-event fluctuations are excluded. In reality, even at fixed impact parameter, the eccentricity could fluctuate for many different reasons: the number of individual nucleons (quarks, partons) participating in the collision could fluctuate, their position in the transverse plane also fluctuate, the multiplicity of produced particles in each of "individual collision" could fluctuate along with deposited energy, etc.. The first serious attempt to calculate the role of such kind of fluctuations for flow analysis in *Monte-Carlo Glauber* model was done by Snellings and Miller [13] who calculate the fluctuation in

$$\varepsilon_{std} = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}, \tag{11}$$

with x, y being the position of the participating nucleons. It was also soon realized, that to take the fluctuations fully into account one should take into account also fluctuations in the direction of the major axes of the overlap region as well as the fluctuation in its center of gravity [17]:

$$\varepsilon_{part} = \frac{\langle y'^2 - x'^2 \rangle}{\langle y'^2 + x'^2 \rangle}, \tag{12}$$

where the eccentricity is calculated relative to the new coordinate system defined by the major axis of the initial system region. The results of the corresponding calculations are presented in Fig. 3. Note significantly larger difference between the standard and participant eccentricities in Cu+Cu collisions compared to Au+Au collisions. Using ε_{part} instead of ε_{std} significantly improves scaling of v_2/ε [17].

With elliptic flow assumed to be proportional to the eccentricity in an event, $v_2 \propto \varepsilon$, flow fluctuations should be determined (at least partially) by the fluctuation in eccentricity. Thus, $v_2\{2\} \propto \varepsilon\{2\}$ and $v_2\{4\} \propto \varepsilon\{4\}$, where

$$(\varepsilon\{2\})^2 \equiv \langle \varepsilon^2 \rangle; \quad (\varepsilon\{4\})^4 \equiv 2(\varepsilon\{2\})^4 - \langle \varepsilon^4 \rangle. \tag{13}$$

The results for $\varepsilon\{2\}$ and $\varepsilon\{4\}$ are also presented in Fig. 3. Two remarks are in order for this plot. The ratio of $\varepsilon\{2\}$'s for Cu+Cu and Au+Au collisions is larger than unity, which, is used, would improve the scaling observed in [17]. Very small values of $\varepsilon\{4\}$ for mid-central and central collisions could complicate the analysis of $v_2\{4\}/\varepsilon\{4\}$ which could become very sensitive to the exact scale of flow fluctuations.
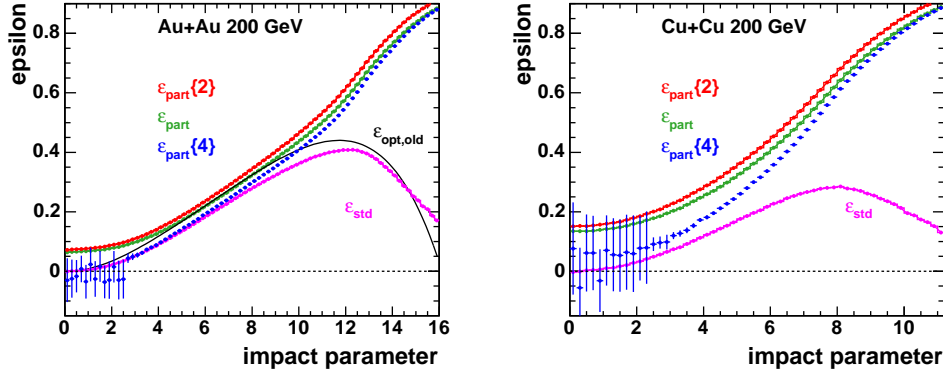

Fig. 3. Eccentricity values for Au+Au and Cu+Cu collisions at $\sqrt{s_{NN}} = 200$ GeV as function of impact parameter. For comparison, the solid line in the left panel shows the results of optical Glauber calculations used in [12, 10].

4. Conclusions

A new method of flow analysis, the Bessel transform of q-distribution has been introduced. It is shown that this method is very similar to the Lee-Yang Zeroes

(using *sum* generating function) and the fitting of q-distribution methods. It was also shown that its sensitivity to flow fluctuations (as well as that of the Lee-Yang Zeroes method) is similar to the four particle cumulant results.

Eccentricity fluctuations relevant for flow measurements $v\{2\}$ and $v\{4\}$ are presented for Au+Au and Cu+Cu collisions at RHIC energies.

Acknowledgments

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